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The Origins of the Gothic Arch and It's Evolution

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Abstract: In this study we tried to reveal the origins of the Gothic pointed arch, how it is transferred from Middle East and developed in Europe. There were two aspects to examine which was of climax importance: the structural and the geometric progression of the pointed arch building techniques. Pointed arch was a symbol of Gothic architecture through ages. This research also aims to demonstrate how Gothic builders modified pointed arch to achieve aesthetic goals with the help of proportion. We wanted to unearth an architectural element's transformation from a vernacular datum to a religious cult. Pointed arch were used long before Gothic building era that its roots may be traced in Eastern civilizations. These civilizations date back to thousands of years from Seljuk to HittiteEmpires. Knowledge of architecture were transferred through wars and migrations which are executed for centuries. That phenomenon was also the same with the pointed arch element. Several soldiers who were recruited as carpenters or masonry craftsmanship in the Crusades, observed the superiority of the pointed arch in Asia Minor and Mesopotamia hinterland's architecture and when they return to their homeland, they used the main feature of Gothic architecture: the pointed arch. Pointed arch became an important architectural element since Gothic builders wanted to erect the cathedrals higher and lighter. The first pointed arch built for erecting tall buildings were carried out by Seljuk Empire architects. Furthermore, weanalysed the adaptation of the pointed arch to Gothic architectureand its development concerning the geometry and structural issues.

Keywords: Pointed Arch, Gothic Building, Proportion

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INTRODUCTION

It is a long-lasting debate to put forth the origins of the Gothic pointed arches acting as windows/doors for openings in walls and ribs for vaults. Gothic architecture has profound techniques in terms of structures which excelled the precedents and altered the building know-how of its era. As architectural historian Banister Fletcher pointed out, Gothic style is remarkable since it appears to represent a complete break with the architectural inheritance of Greece and Rome (Fletcher, 1987). He argues that the Gothic builders felt liberated on discarding the architectural elements formerly used. The most of architectural characteristic of Gothic is grew out of Romanesque period. The derivations followed each country's cultural and religious attitudes. Without a doubt the pointed arch existed in

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Asia Minor (Anatolia) and Mesopotamia before it was built in Europe. There are considerable number of researchers who think pointed arch was transferred from East to West.

According toarchitect and theoretician Christopher Wren, the existence of the pointed arch is antecedent to the Crusades. Where Wren said it, it found the echoes of Gothic arch in Eastern settlements especially in Seljuk Empire established between 11th and 14th century. Wren was sure that Islamic architects used the pointed arch as a decisive form, and he evolved the theory of the Eastern origin of the Gothic style. Pointed arch existed long time before, archaeologist Erich Schmidt, said to be the first researcher who raised attention to the pointed arch that it was first built by Sassanids in Damghan, Iran in circa 700 (Briggs, 1993); but Seljuk architects made it astriumphal by building huge versions especially in Anatolia in circa 11th century. Architectural Historian Otto von Simson notes that "The Crusades affected the development of the Gothic style. Crusaders returning from the Holy Land, brought with them many relics, and church fathers wanted to display these holy objects." (Simson, 1988).

Likewise, theorist Alessandro Camizpointed out that, since the Crusaders migrated to Cyprus from Jerusalem (in circa 1190) they transferred the building techniques what Early Gothic architecture inherited from Holy Land and flourished in Cyprus as an important experimentation phase before it was imported to Northern Europe. He stresses on the point that the church of Saint Anne in Jerusalem was built 13 years ago from the cathedral of Saint Denis (1144) in Paris featuring all the pointed arches, groin vault and polylobate pilasters. He concludes that the Crusaders created an era of architectural style in Cyprus coined as "Transitional Gothic". The Cathedral of Saint Sofia (in circa 1190) (present-day Selimiye Mosque) is a good example for this. There is another unique opinion that Gothic pointed arch is brought to Europe by Moors to Spain (Young, 1789). We should also be alert not to overlook the vernacular architecture in Mesopotamia. Ferwati and Mandour (2008) reported that, there are 25 different pointed arch types in Damascene courtyard houses in Syria.

THE GROWTH OF POINTED ARCH IN EARLY ISLAMIC ARCHITECTURE

Islamic Architecture flourished after prophet Mohammad's arrival. There was a tendency towards civilization and prosperity in daily life. According to Warren (1991) the dynasty of Umayyads preferredusing slightly pointed arch rather than a semi-circular arch in the 7th century. It was a two-centred arch (drop arch) of which its base was divided into 6. By time, the base was divided up to 10. Geographically Arabs had only sun-dried mud bricks to build in adobe construction. When they had to build mosques with greater spaces, they might have needed a stronger architectural element than semi-circular arch. Like Erich Schmidt, architectural historianKeppel Archibald Cameron *Creswell* reported that the Friday Mosque's pointed arches in Damghan was the first examples of its kind (Warren,1991)(Briggs, 1993). Scholar Peter Draper (2005) notes that after Umayyad rule, Abbasids came to government, they built pointed archesfrom bricksin the 9th century. This new building technique gained recognition in Egypt and other countries since it was superior to semi-circular arch in load-bearing function. When Seljuk Turks ruled over all the Islamic region, they also used pointed arch, with one difference; they implemented the pointed arch with stone.

EARLY GOTHIC ARCHIN EUROPE

The Early Gothic architecture suggested by the art historian Henri Focillon be defined as "*The Romanesque of the Ile-de-France*" (Simson, 1952).In general, the change from semi-circular to pointed arches came to the region late, taking place earlier in chancel arches and arcades than in doors and windows, which were of less structural importance. Saint Denis basilica (1135) was the earliest

Gothic church in Europe built in Ile-de-France. It was patronned by Capetians dynasty during 11th century and had highly influences of Romanesque antecedents. Capetian Monarchy sent soldiers and carpenters for Crusades to Anatolia (Naus, 2016). They were attracted by the Seljuk Empire architecture and its arches. A pioneer architect and researcher Christopher Wren, put forward that Islamic architects (alluding Seljuk Empire architects) used the pointed arch to build higher, and he evolved the theory of the Saracenic origin of the Gothic style(Naus, 2018).

Several churches in Europe had extensions and thus one can see both semi-circular and pointed arches at the same time in one building complex. According to Gall, the crossing vault of Saint Trinité in Paris (in circa 1000) was built last of all, when the adjacent parts of the building could take the thrust. In its interior, where the nave meets the crossing, the beginnings of a round arch have been preserved, standing at the same level as the three original semi-circular arches of the crossing. The central part of this round arch was pulled down, flat responds were added to the piers, and then a pointed arch was built -possibly the first ever to have been built in connexion with a rib-vault(Gall,1925).

With the help of pointed arches, the vaults were built more reliably. To give ribs the form of the pointed arch was an innovation significant in several ways. It made the construction of a vault completely independent of the shape of the bay in plan (Wilson, 1990). It also brought complete conformity to the arches of a building. There are 3 main types of pointed arches created by geometric interpretations: Drop, Lancet and Equilateral. We will be referring to these pointed arches used especially by both Seljuk and Gothic architects.

THE SELJUK EMPIRE IMPACT ON POINTED ARCHES

When Seljuk Turks started ruling Anatolia and her hinterland (Iran, Iraq, Armenia and Syria) starting from the 10th century, they established a civilized empire cultivated with science and technology. They invited countless mathematicians, architects, and philosophers to their young empire. After that, they built government buildings including astronomy research centres, hospitals, hotels for the voyagers and merchants (kervansarai), universities and mosques in every city and town. The Seljuk architects were famous for their huge gates as these buildings resembled their mighty empire like the triumphal arches of Roman Empire. In these portal-like gates (tachkapy) they have interpreted pointed arch which helped to build higher and higher.

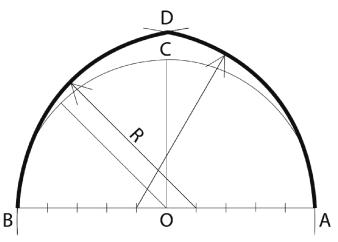


Figure 1: Drop Pointed Arch which is used in Seljuk Empire (known as siege arch), dividing the base into 10 segments, BCA is semi-circular arch and BDA is Seljuk Empire Arch, According to Art Historian Orhan Cezmi Tuncer (Ayan, 2012)

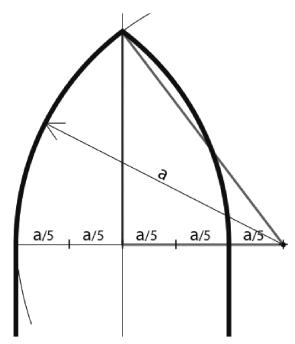


Figure 2: Lancet Pointed Arch used in Europe

These huge gates with the porch, first built in the 11th century, supported by semi-circular arches, formed almost a vault for the threshold. Since the load is too much for this arch, they designed another arch on top of it which was pointed one. It was called *siege* arch (In Turkish *kushatma kemeri* also known as *kavsara*) (Bulut, 2019). This pointed arch support has brought the possibility of building a muqarnas. They have discovered that pointed arch was superior to semi-circular arch. So, conventionally used semi-circular arc was replaced by pointed arch in all the aspects of architecture and engineering applications. The *tachkapy* had a rectangle shape in the order of 4/5. Figure 1 shows the Seljuk Empire Pointed arch (also known as *siege arch*) which is used widely throughout the Empire. This *droptype* pointed arch has the height/base ratio of 3/5(Ayan, 2012). Interestingly it is coined as 3/5 *arch* in European Gothic applications. The tachkapy of the most famous building is Divrigi Mosque and Hospital in Sivas (1220), Türkiye is made from this arch. Figure 2 shows a *lancet type of* arch which is used widely in Gothic churches and cathedrals. It is quite higher than 3/5 drop arch of Seljuk as seen in Figure 1. It has a 1/1 ratio.

Art historian Fugen Ilter notes that Seljuk engineers and architects have built stone bridges in the form of pointed arches (Ilter, 1993). This building technique enabled the bridge to function without a risk of demolition against heavy camel caravans, seismic movements and floods.

STRUCTURAL IMPROVEMENT OF ARCHES

Gothic architects used empirical building constructions. Before, there was a factoid "law of similitude": a valid structural form continues to be correct independently of its size. Galileo argued the impossibility of the existence of this kind of principle. In structures supporting as the main load their own weight the dead load rises as the cube of the linear dimensions while the section of the structural members rises as the square; therefore, the stresses rise linearly with the size (Huerta, 2006). In masonry structures, the compressive load must be internally balanced as well. As state of the art, strength scales with cross sectional area instead of volume.

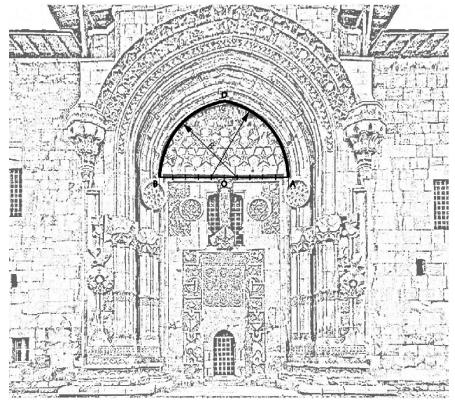


Figure 3: The Seljuk Empire Tachkapy Supported by a Siege Pointed Arch (Coined as Drop Arch),
Divrigi Mosque and Hospital in Sivas, Turkiye

French architect and author Viollet-le-Duc defined "elasticity" as the essential quality of the Gothic arch, being the skeleton of a constructive system where "everything is equilibrium and, consequently, movement" (Viollet-le-Duc, 1854-1868). As the Mainz architect Johannes Wetter explained that the dissolution of the wall was the product of four main factors when he analysed Mainz Cathedral (Rebuilt in circa 1200). "...- the cross-vault, the rib, the pointed arch, and the buttress- this principle has always been emphasized as one of the main characteristics of the Gothic style. The pointed arch enhances the verticality of the thin layers of space produced by the ribs..." (Frankl, 1962). So, for him, the pointed arch was an integrating element of the overall structure and represented verticality (Schwarzer, 1993).

Aita et al. (2019) addressed the mechanical behaviour on collapse and failure with the same span under their own weight which showed the superiority of pointed arch to the semi-circular arch. In semi-circular arch, oblique lateral thrust is towards approximately 45 degrees off the load bearing wall. In the pointed arch the thrust is almost perpendicular with the ground.

All the stones in a pointed arch are in compression; bending stress is eliminated therefore they behave like a monocoque body (Sandaker, Eggen and Cruvellier, 2011). Several architects and engineers have studied this phenomenon throughout history. One of them is Leonardo da Vinci. He analysed and illustrated how typical arches act against failure based on outward thrust. The other important figure is Antonio Gaudi. Gaudi invented the Catenary arch which is superior to pointed arch in structural means. Catenary was a static arch which was originated from ad quadratum (Osserman, 2010).

As one monitors in Turkey Anatolia may find a greatvariety of arches especially in Cappadocia caves. Before the Constantine's adoption of Christianity as the religion of the Empire. Since the

early Christians were hiding themselves in caves, they built hidden churches carved from tuff stonein Cappadocia easily. We experienceeven the catenary arches in Cappadocia.

In fact, millenniums ago there was another civilization established the city walls and her architecture with arches, The Hittite Empire. The Hittite Portal Arch is anenhanced corbelled arch. The Hittite gate stones were constructed by offsetting horizontal courses of stone. Abutments on sides are due to make the portal gate more stable against lateral loads. It is interesting to note that The Hittite Portal Arch is approximately in a catenary arch form. When superimposed it is between the pointed arch and catenary form. Scholars likeAlessandra Romano and John A. Ochsendorf studied the minimum thrust on pointed arches which copes with the Hittite arch in Turkey (2010).

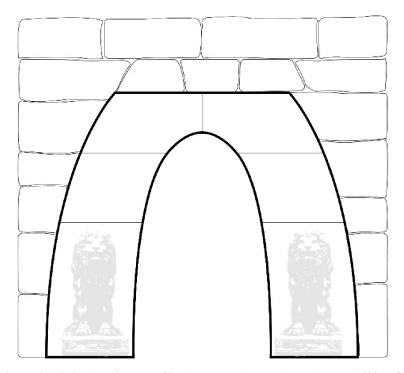


Figure 4: Hittite Architecture City Entrance Portal Arch, Turkey, 1600 B.C.

Seljuk Empire's architectsbuilt arches as pointed arches (when superimposed)which are approximately on curved line of thrust. In this geometry, the horizontal component of line of thrust becomes minimum(Lancaster, 2005). Thus, builders did not need to implement a tie rod. The semi-circular arch needs tie rod connection not to fail. (Iron tie rods have been used for centuries as auxiliary and as reinforcing elements for arches.)

In Figure 5 the thrust line passing through a semi-circular arch proposes a pointed arch. Geometrical estimations of the structural elements were quite in demand amongst the Gothic architects. Derand divides the semicircle into three. By using the intersection line, he supposes the thickness of the buttress carrying the arch (Huerta, 2012).

Gothic builders developedtas-de-chargewhich is a technique for carving the stones of pointed archesto support the vault structure in a topology optimization manner. This idea might be an inspiration coming from Seljuk Empire architecture's muqarnas that used to thin the load bearing shaft of upper stone rib work in the vaults. This technique also links the arches sitting on columns enabling slenderness in structural means and a sense of fragility of architectural elements (Fehér,

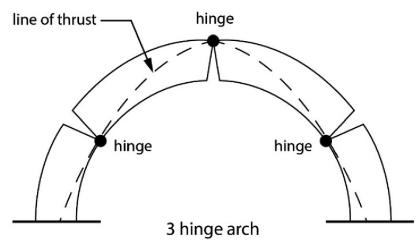


Figure 5: The shape of the 'line of thrust' resembles pointed arch form (Jean Rondelet, 1812)

2021). Pointed arch, in the form of tas-de-charge was combined with the vault at a much later date. According to a Gothic architecture scholarRobert Branner (1961), tas-de-charge was likely first applied in the Cathedral of Chartres's (1145) nave vaults as early as around 1205. Fehér (2021) argued that the reason behind the performance of Gothic arch the height has been enhanced by the Tas-de-charge method. The configuration of ceiling structures showed growth from Groin Vault to Ribbed Vault. This phenomenon brought a skeleton look on load bearing elements.

GEOMETRY, MEANING AND AESTHETICS IN GOTHIC ARCH

When chronologically analysed it is obvious that Gothic architecture has no standard rules in terms of geometry and proportion. It differs through country to country and even every individual building. According to Bork, by the mid of 12th century, draftsmanwas responsible for designing the buildings. He puts forward that Gothic architecture was governed by procedural conventions, rather than by fixed canons of proportion (Bork, 2016). Gothic builders wanted to achieve a slender look and surpass Renaissance semi-circular arch capabilities. Since the semi-circular arch buttress design were held with geometric calculations the thickness of the buttresswas quite thick and looking chubby. Thus, Gothic arch proportions was altered by the designer's own vision and reached the aesthetic standards formed by structural optimismIn Gothic churches, structural members are not sized according to spatial proportioning formulations (Sanabria, 1982).

As Huerta puts forward that, Gothic builders did not have any scientific calculation for structural issues instead they had *scientia* which is directly related with geometric composition. Scientia eases to design the loadbearing system proportionally and define the thickness of the walls carrying the superstructure according to the span. The masters of the Gothic cathedrals reached the climax by minimum trial and error method. By taking the advantage of proportionally balanced solids and voids, builders were successful in trying higher structures (Huerta, 2012).

As stated by Rivin and Toporov (2000), the construction of the longitudinal and cross sections of several Gothic cathedrals were formed by *graphostatic* method. Thus, the arches on the façades tracing the line of thrust were determined by this way. This arch has a ratio of (height to base) 3/5 which refers to drop arch.

In his widely read book The Gothic Cathedral, Otto von Simson argued that early Gothic builders were strongly influenced by Platonic ideas of geometrical and musical harmony, and that their designs

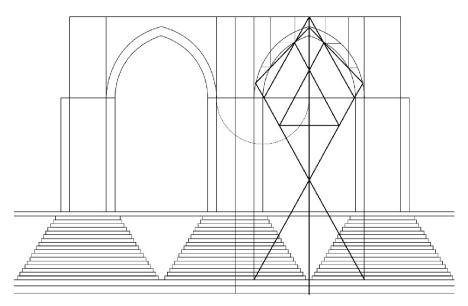


Figure 6: Graphostatic method to determine the pointed arch in Gothic architecture

for structures such as Chartres Cathedral were based on ideal figures such as the equilateral triangle, square, pentagon, and the so-called Golden Rectangle (Simson, 1956).

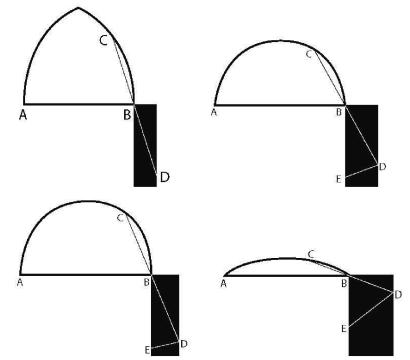


Figure 7: Geometrical correspondence of abutments rule, fromDerand (1643)

There was another method to design an arch inherited from Roman Empire: Vesica Piscis. Vesica Piscis is the main symbol of creation seen almost in all the oval creatures. We find its roots in the nature when we analyse the geometric growth. Vesica Piscis' geometric development were used as analogical approach to symbolize the womb of a woman which may give birth. In Christianity, the fish, represented by a Vesica Piscis-like graphic construction, was the symbol of Jesus of Nazareth (Barallo

et al. 2015). Correlatively this simple shape can be proliferated into more complex forms. Gothic builders also used Vesica Piscis but rather for determining the proportions of the arch and vaults. This type of arch is called *Equilateralarch* and has a ratio of 5/6.

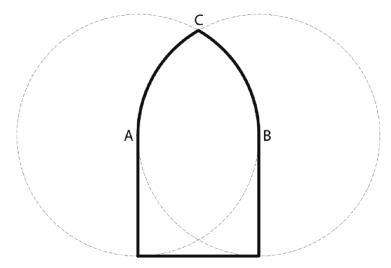


Figure 8: Vesica Piscis forming a pointed arch, called as Equilateral Arch

There were other methods of formations like Ad Quadratum, and it is that one takes a square and draws a diagonal square inside and in a series of further steps gives the technique for "elevating" a cathedral pinnacle. This general Ad Quadratum technique was used to produce other structures including the cross-sections of buildings. In Milan Cathedral building elevation, there used to be a geometrical scheme in which a regular hexagon, an inverted square and a triangle has been inscribed within a circle.

Gothic architecture has always been a dynamic building process regarding the will of perfection in terms of optimum structure and material usage. It was rather a building experiment to build higher and lighter. This was the main point which Gothic architecture differed from Classical periods. Instead, it was never completely broken, it became attenuated (Padovan, 1999). From Goethe's and Villard de Honnecourt's early writings on Gothic builders, we understand that they were also involved in sequences of geometrical operations. Geometry ensured some kind of standardization. It was both for a fixed analogy (symbolism and aesthetics) and modular constructional functions. In Fredrik Macody Lund's Ad Quadratum book, a system based on the pentagram is imposed on the cross-section of Notre-Dame in Paris. As stated by Frankl (1962) the overall complexity of European Gothic buildings was the result of the milieu of medieval stonemasons whose know how was generated in the workshops. Reims Cathedral's pointed arches were constituted by dividing the base into 5 as a drop arch type. This drop arch type has a ratio 3/4 and slightly higher than Seljuk Empire Arch. Figure 9 and 10 demonstrates how this type of arch is formed and applied on the cross section.

Dimitris Theodossopoulos (2008) argues that English Gothic arches are more stable and static when compared to French precedents. He draws attention on Durham Cathedral that it is a unique Gothic example in Europe, since it has the successful application of ribs supporting the barrel vaults.

There are several hypotheses on Milan Cathedral's (1386) generation of pointed arches. Essential heights of the cathedral are derived with the help of equilateral triangles inscribed within a circle. These triangles at first is used to constitute the arch. The Milanese building culture, nevertheless, had

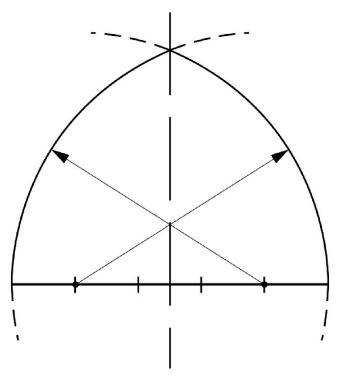


Figure 9: Drop Pointed Arch developed by Gothic Architects also known as quinto acuto

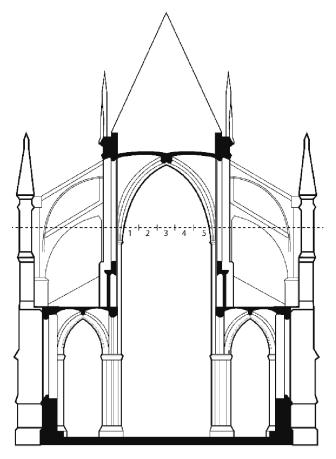


Figure 10:Drop PointedArch (Villard's drop arch also called *quinto acuto* arch) is formed by dividing the base into 5 segments, Reims Cathedral Cross Section, France, in circa 1200.

a strong influence on discussions of the Gothic approach to geometry, especially since they provide a frequently cited source for the phrases "ad quadratum" and "ad triangulum." (Sebregondi et al., 2016). In Milan, as Zizi et al. have reported, Lancet, Equilateral and Drop pointed arches were built between 12th and 13th centuries (Zizi et al., 2021).

Likewise, Strasbourg Cathedral (1300) in Germany were designed so that every element of the design emerges naturally from a simple process of geometrical development based on the unfolding of forms from the square and triangular "seeds" (Bork, 2016).

In high Gothic era, French carpenter-architect Villard De Honnecourt, introduced several pointed arches which are enrolled in his sketchbook. His sketchbooks were used as manuals for builders. He travelled to record the medieval religious buildings throughout Europe. He noted the building processes of both semi-circular and pointed arches. He established a set of rules to construct with proportions with the help of a compass (Bechmann, 1988) (Huber, 2009).

CONCLUSION

There are many opinions come to consensus that pointed arch building techniquewas transferred through Europe after Crusades executed in Anatolia and Mesopotamia. It is likely that the Gothic architects sought ways to build higher religious architecture like Seljuk Empire architects. There was only one way to make the load bearing building higher than the precedents: it was the pointed arch. Some applications were acquired from Seljuk Empire Arches like the 3/5 arch.

It is interesting to note that in Gothic Style, architects still used geometric developments by using ad quadratum and ad Triangulum methods. They wanted to preserve the ancient proportions while building higher which goes beyond the standards of all times. Gothic architects proceeded using geometry to ensure standards.

Gothic arches enable an aesthetic and structural articulation. Gothic architects used pointed arch to reach a level ofperfection in terms of optimum structure and material usage. It was rather a building experiment to build higher and lighter. In the high Gothic era, the stone was cut due to structural requirements and several techniques like tas-de-charge has been evolved to carry excessive loads. This phenomenon led the way to skeletal structures in following centuries.

Consequently, pointed arch was superior to semi-circular arch in terms of loadbearing. It enabled to build slender abutments. It also liberated the architect from the plan outline. The pointed arch enabledconstruction of the of the walls inthin layers.

Pointed arch became another feature in architectural elements that it lubricated the will of perfection in building technology. With the help of pointed arch architects overcome the structural challenges while building higher. We can conclude that during the conveyance of pointed arch throughout continents, the scholarship of proportion was not preserved but rather the structural knowhow was cited and improved.

As a matter of fact, this achievement in architectural history was due to the interaction of Eastern and Western civilizations. This brought a co-creation in the pointed arch topology. The difference was the Eastern civilizations (like Seljuk Empire) used the pointed arch for praising a variety of its governmental buildings, while the Western civilizations used it for only religious buildings. Therefore, we may say that pointed arch has been transposed from a structural architectural element into a religious cult in Europe during the Gothic era in symbolic terms.

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